

A GENERAL RELATION BETWEEN FREQUENCY AND DURATION OF PRECIPITATION

LEONARD L. WEISS

U.S. Weather Bureau, Washington, D.C.

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ABSTRACT

A single, simple algebraic equation is given expressing the complete frequency-duration relationship of the precipitation regime at any station. The constants involved in the equation are theoretically fixed for the frequencies. Those for the durations are based on empirical determinations. The equation readily lends itself to use in high-speed computer programs to calculate the entire array of most usually desired frequency-duration values.

The entire frequency-duration precipitation regime at any station may be represented by a geometric surface in three dimensions, $z = F(x, y)$, where z is the precipitation amount corresponding to the frequency represented by a linearized variate x and the duration represented by a linearized variate y , as illustrated in figure 1.

Under the requirement that it be a linear function of either variate when the other is constant, z must be of the form

$$z = Ax + Bxy + Cy + D \quad (1)$$

from which it follows by differentiation:

$$\frac{\partial z}{\partial x} = A + By \quad (2)$$

$$\frac{\partial z}{\partial y} = Bx + C \quad (3)$$

$$\frac{\partial^2 z}{\partial x \partial y} = B \quad (4)$$

Then the evaluation of (1), (2), (3), and (4) at $x_1 = 0$, $y_1 = 0$ gives respectively (referring to fig. 1):

$$A = \left(\frac{\partial z}{\partial x} \right)_1 = \frac{Z_3 - Z_1}{x_2} \quad (5)$$

$$B = \left(\frac{\partial^2 z}{\partial x \partial y} \right)_{1,1} = \frac{(Z_4 - Z_3) - (Z_2 - Z_1)}{x_2 y_2} \quad (6)$$

$$C = \left(\frac{\partial z}{\partial y} \right)_1 = \frac{Z_2 - Z_1}{y_2} \quad (7)$$

$$D = Z_1 \quad (8)$$

Substitution of the right sides of (5), (6), (7), and (8) into (1) then gives:

$$z = \left(\frac{Z_3 - Z_1}{x_2} \right) x + \left[\frac{(Z_4 - Z_3) - (Z_2 - Z_1)}{x_2 y_2} \right] xy + \left(\frac{Z_2 - Z_1}{y_2} \right) y + Z_1 \quad (9)$$

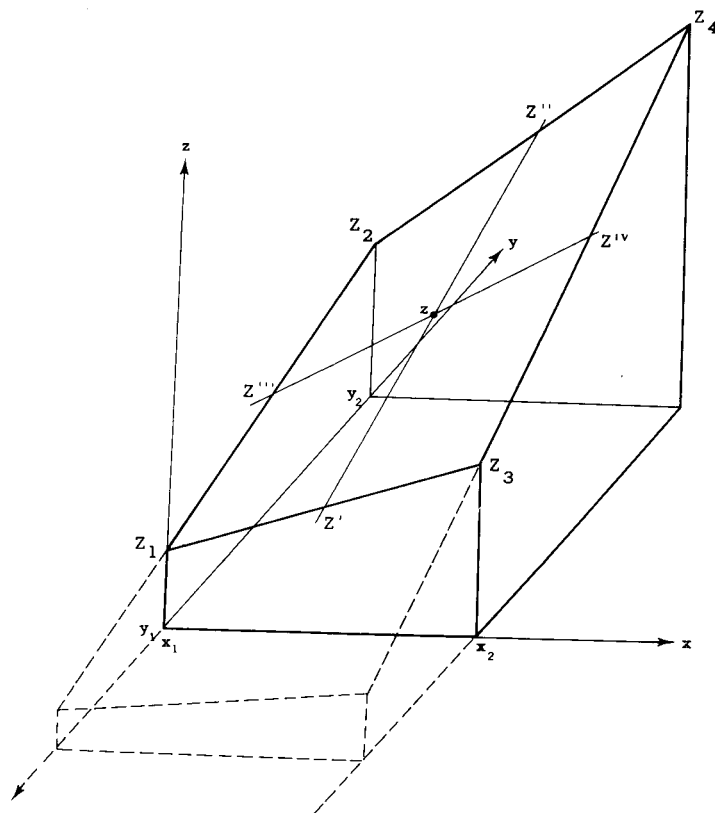


FIGURE 1.—Schematic representation of the frequency-duration precipitation regime as a geometric surface in three dimensions expressed by equation (9).

TABLE 1.—Values of linearized frequency variate for partial duration series

Frequency (years)	1	2	5	10	25	50	100
x	-6.93	0	9.2	16.1	25.3	32.1	39.1

The linearized frequency variate for partial duration series data is given in table 1. These values are based on the theoretical analysis of Chow [1] from which it follows that $\ln P_a = -1/T_p$, where T_p is the return period for the partial duration series, and $x = -\ln \ln (1/P_a)$. The values of x corresponding to the probability P_a are from tables published by the National Bureau of Standards [2].

A linearized duration variate y , can be related to the duration t (in hours) by requiring that

$$t_y^n = 0.01 [(24)^n - 1] y + 1 \quad (10)$$

where the y scale is selected so that $y=0$ corresponds to $t=1$ hour and $y=100$ corresponds to $t=24$ hours. Empirical observations have shown that the 6-hour value is very closely the mean of the 1-hour and 24-hour values. From this it follows that n is very nearly $\frac{1}{6}$. Equation (10) then becomes:

$$t_y^{\frac{1}{6}} = 0.00698 y + 1 \quad (11)$$

from which table 2 is determined. These values have been tested with data from first-order Weather Bureau stations and found to be applicable quite generally.

When the values of $x_2=39.1$ and $y_2=100.0$ are chosen from tables 1 and 2 and inserted into equation (9) the complete frequency-duration relationship is:

TABLE 2.—Values of linearized duration variate for various durations

Duration t (hours)	.5	1	2	3	6	12	24
y	-15.6	0	17.6	28.8	49.9	73.4	100.0

$$z = 0.025575 (Z_3 - Z_1)x + 0.00025575 [(Z_4 - Z_3) - (Z_2 - Z_1)]xy + 0.01(Z_2 - Z_1)y + Z_1 \quad (12)$$

For any station for which are available the four partial-duration series precipitation values, Z_1 , Z_2 , Z_3 , and Z_4 , listed below (the ones most usually available), equation (12) can be solved for any desired combination of frequency and duration. If other duration values must be used, appropriate changes must be made in equation (12).

Z_1 =the 2-year, 1-hour partial-duration series value

Z_2 =the 2-year, 24-hour partial-duration series value

Z_3 =the 100-year, 1-hour partial-duration series value

Z_4 =the 100-year 24-hour partial-duration series value.

Equation (12) is particularly useful in programing high-speed computers to calculate the entire array of usually desired frequency-duration values.

REFERENCES

1. Ven Te Chow, "Discussion of 'Annual Floods and the Partial-Duration Flood Series' by W. B. Langbein (T.A.G.U., 1949, pp. 879-881)," *Transactions of the American Geophysical Union*, vol. 31, No. 6, Dec. 1950, pp. 938-941.
2. U.S. National Bureau of Standards, "Probability Tables for the Analysis of Extreme-Value Data," *Applied Mathematics Series* 22, July 6, 1953.